Off-diagonal structure of neutrino mass matrix in see–saw mechanism and electron–muon–tau lepton universality

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By a simple extension of the standard model in which $(e-\mu-\tau)$ universality is not conserved, we present a scenario within the framework of see–saw mechanism in which the neutrino mass matrix is strictly off–diagonal in the flavor basis. We show that a version of this scenario can accomodate the small angle MSW solution of the solar neutrino problem and $\nu_\mu-\nu_e$ oscillations claimed by the LSND collaboration. Another version accomodates atmospheric $\nu_\mu-\nu_\tau$ oscillations and large angle solution in solar neutrino experiments.

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The minimal standard model involves three left-handed neutrino states and as such does not admit renormalizable interactions that can generate neutrino masses. However, there are three sets of data which suggest neutrino oscillations, and hence that neutrinos have mass. One is thus forced to look for viable extensions of the standard model. Before we consider one such extension, let me summarize the three set of data [1–8] which claim neutrino oscillations:

a) Atmospheric neutrnio experiments [4–6] [oscillations of ν_μ probabley into ν_τ]

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{23} \simeq (2-6)\times 10^{-3}\ \text{eV}^2$$
$$\sin^2 2\theta_{23} \equiv \sin^2 2\theta_1 \simeq 0.82 - 1.0$$

b) LSND collaboration for $\nu_\mu-\nu_e$ oscillations [7,8]

$$0.3\ \text{eV}^2 \le \Delta m^2_{12} \le 2.0\ \text{eV}^2$$
$$10^{-3} < \sin^2 2\theta < 4\times 10^{-2}$$

c) Solar Neutrino Experiments [2,3]

$$\text{MSW:}\quad \text{SMA:}\quad 3\times 10^{-6}\ \text{eV}^2 \le \Delta m^2_{23} \le 10^{-5}\ \text{eV}^2$$
$$2\times 10^{-3} < \sin^2 2\theta_s < 2\times 10^{-2}$$

or

$$\text{MSW:}\quad \text{LMA:}\quad 10^{-5}\ \text{eV}^2 \le \Delta m^2_{12} \le 10^{-4}\ \text{eV}^2$$
$$0.6 \le \sin^2 2\theta_3 \le 0.95$$

or

$$\text{V0:}\quad 5\times 10^{-11}\ \text{eV}^2 \le \Delta m^2_{12} \le 5\times 10^{-10}\ \text{eV}^2$$
$$0.6 < \sin^2 2\theta_3 < 1.0$$

The three different mass splittings Δm^2 in (a), (b) and (c) above seem do not compatible with a mixing of only three neutrino flavors.

It is known that there are two main mechanisms to generate tiny neutrino masses [9]. One is the see-saw mechanism [10] requiring the existance of superheavy $\left(\ge 10^{10}\ \text{GeV}\right)$ right handed Majorana neutrinos while in the other tiny masses are [11–13] generated through higher order loop effects. Both require an extension of the standard model and in both senarios light neutrinos are Majorana.

A well known example of the second mechanism is the Zee model [11] in which the neutrino mass matrix is strictly off-diagonal in the (e,μ,τ) basis. The purpose of this paper to revive a modest extension of the standard model in which neutrinos have small masses and lepton flavor $[(e,\mu,\tau)$ universality$]$ is not conserved. It was shown [14] that that this extension within the framework of see–saw mechanism also lead to off-diagonal neutrino mass matrix in flavor basis. One version of the Zee model can accomodate mass hirarchy $|m_1| \simeq |m_2| \gg |m_3|$, $\Delta m^2_{13} \simeq \Delta m^2_{23} \simeq \Delta m^2_{\text{atm}}, \Delta m^2_{12} \simeq \Delta m^2_{\text{solar}}$, compatible with both atmospheric and solar neutrino data, the latter has to be described by large angle vacuum or MSW oscillations [15]. We show that in contrast a version of our model accomodates mass heirarchy $|m_3| \simeq |m_2| \gg |m_1|$, $\Delta m^2_{12} \simeq \Delta m^2_{13} \simeq \Delta m^2_{\text{LSND}}$ and $\Delta m^2_{23} \simeq \Delta m^2_{\text{solar}}$ for small angle MSW oscillations. However, there is another version of our model where one gets the same results as in the Zee model with $|m_1| \simeq |m_2| \gg |m_3|$, $\Delta m^2_{23} \simeq \Delta m^2_{13} \simeq \Delta m^2_{\text{atm}}, \Delta m^2_{12} \simeq \Delta m^2_{\text{solar}}$.

I. RESTRICTIONS ON NEUTRINO MIXING ANGLES

Let us consider an off-diagonal Majorana mass matrix in (e, μ, τ) basis

$$M = m_0 \begin{pmatrix} 0 & a_{e\mu} & a_{e\tau} \\ a_{e\mu} & 0 & a_{\mu\tau} \\ a_{e\tau} & a_{\mu\tau} & 0 \end{pmatrix} \tag{1}$$

It is convenient to define neutrino mixing angles as follows

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{2}$$

where

$$U = \begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 e^{i\delta} \\ -c_1 s_3 - s_1 s_2 c_3 e^{i\delta} & c_1 c_3 - s_1 s_2 s_3 e^{i\delta} & s_1 c_2 \\ s_1 s_3 - c_1 s_2 c_3 e^{i\delta} & -s_1 c_3 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 \end{pmatrix} \tag{3}$$

with $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. We shall put$\delta = 0$. Due to the off-diagonal structure of the mass matrix (1), the following relations are derived [13]:

$$m_2 = -\frac{\cos^2\theta_3 - \tan^2\theta_2}{\sin^2\theta_3 - \tan^2\theta_2} m_1, \;\; m_1 + m_2 + m_3 = 0 \tag{4}$$

$$\cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 = \frac{1}{2} \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \left(3\cos^2\theta_2 - 2\right) \tag{5}$$

$$-\sin 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 - \frac{1}{2} \cos 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \left(3\cos^2\theta_2 - 2\right) = a_2 a_{\mu\tau} \tag{6}$$

$$-\cos 2\theta_3 \sin\theta_1 \sin\theta_2 \cos\theta_2 + \frac{1}{2} \cos\theta_1 \sin 2\theta_3 \cos\theta_2 \left(3\cos^2\theta_2 - 2\right) = a_2 a_{e\mu} \tag{7}$$

$$-\cos 2\theta_3 \cos\theta_1 \sin\theta_2 \cos\theta_2 - \frac{1}{2} \sin\theta_1 \cos\theta_2 \sin 2\theta_3 \left(3\cos^2\theta_2 - 2\right) = a_2 a_{e\tau} \tag{8}$$

where

$$a_2 = \frac{m_0}{m_2} \left(\cos^2\theta_3 \cos^2\theta_2 - \sin^2\theta_2\right). \tag{9}$$

We also give here the transition probabilities

$$\begin{aligned} P\left(\nu_\mu - \nu_\tau\right) = [&-\frac{1}{4}\sin^2 2\theta_1 \sin^2 2\theta_3 \left(1 + \sin^2\theta_2\right)^2 + \sin^2 2\theta_1 \sin^2\theta_2 \\ &+ \cos 2\theta_1 \sin 2\theta_1 \sin 2\theta_3 \cos 2\theta_3 \sin\theta_2 \left(1 + \sin^2\theta_2\right) + \cos^2 2\theta_1 \sin^2 2\theta_3 \sin^2\theta_2] \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right) \\ &+ \sin 2\theta_1 \cos^2\theta_2 \left[\left(\sin 2\theta_1 \cos^2\theta_3 - \sin 2\theta_1 \sin^2\theta_2 \sin^2\theta_3 + \sin 2\theta_3 \cos 2\theta_1 \sin\theta_2\right) \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right)\right. \\ &\left. + \left(\sin 2\theta_1 \sin^2\theta_3 - \sin 2\theta_1 \sin^2\theta_2 \cos^2\theta_3 - \sin 2\theta_3 \cos 2\theta_1 \sin\theta_2\right) \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)\right] \end{aligned} \tag{10}$$

$$\begin{aligned} P\left(\nu_e - \nu_\mu\right) = &\left[\sin^2 2\theta_3 \cos^2\theta_2 \left(\cos^2\theta_1 - \sin^2\theta_1 \sin^2\theta_2\right) + \sin 2\theta_1 \sin 2\theta_3 \sin\theta_2 \cos^2\theta_2\right] \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right) \\ &+ \sin 2\theta_2 \sin\theta_1 \left[\left(-\cos\theta_1 \cos\theta_2 \sin 2\theta_3 + \sin\theta_1 \sin 2\theta_2 \sin^2\theta_3\right) \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right)\right. \\ &\left. + \left(\cos\theta_1 \cos\theta_2 \sin 2\theta_3 + \sin\theta_1 \sin 2\theta_2 \cos^2\theta_3\right) \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)\right] \end{aligned} \tag{11}$$

$$P\left(\nu_\mu-\nu_\tau\right) = \left[\sin^2 2\theta_3 \cos^2\theta_2 \left(\sin^2\theta_1 - \cos^2\theta_1 \sin^2\theta_2\right) - \sin 2\theta_1 \sin 2\theta_3 \cos 2\theta_3 \cos^2\theta_2 \sin\theta_2\right] \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$
$$+ \sin 2\theta_2 \cos\theta_1 \left[\left(\sin 2\theta_3 \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin^2\theta_3 \sin 2\theta_2\right) \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right)\right.$$
$$\left. + \left(-\sin 2\theta_3 \sin\theta_1 \cos\theta_2 + \cos\theta_1 \cos^2\theta_3 \sin 2\theta_2\right) \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)\right] \tag{12}$$

$$P\left(\nu_e-\nu_e\right) = 1 - \cos^4\theta_2 \sin^2 2\theta_3 \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right) - \sin^2 2\theta_2 \sin^2\theta_3 \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right) - \sin^2 2\theta_2 \cos^2\theta_3 \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right) \tag{13}$$

It may be noted that

$$\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = m_2^2 - m_1^2 + m_3^2 - m_2^2 + m_1^2 - m_3^2 = 0 \tag{14}$$

II. EXTENSION OF THE STANDARD MODEL AND NEUTRINO MASS MATRIX

By a simple extension of the standard electroweak gauge group to

$$G \equiv SU_L(2) \times U_e(1) \times U_\mu(1) \times U_\tau(1),$$

it was shown [14] that the Majorana masses for light neutrinos are generated through diagrams shown in figure 1.

Here $\phi^{(i)}$ and $\Sigma^{(i)}$ are respectively three $SU_L(2)$ Higgs doublets and singlets with appropriate $U_i(1)$ quantum numbers; h's and f's are the corresponding Yukawa couplings. The symmetry is spontaneously broken by giving vacuum expectation values to Higgs bosons $\phi^{(i)}$ and $\Sigma^{(i)}$: $\left\langle\phi^{(i)}\right\rangle = \frac{v_i}{\sqrt{2}}$ and $\left\langle\Sigma^{(i)}\right\rangle = \frac{\Lambda_i}{\sqrt{2}}$. For simplicity we shall take $v_1 = v_2 = v_3 = v$ and $\Lambda_1 = \Lambda_2 = \Lambda_3 = \Lambda$ (any difference can be absorbed in the corresponding Yukawa couplings h and f). We take $\Lambda \gg v$ so that $X-$bosons which break the $e-\mu-\tau$ universality as well as the the Majorana mass term for heavy neutrinos N's are superheavy. In order to simplify the calculation, we put $f_{12} = f_{13} = f_{23} = f$ (any differences can again be absorbed in h-couplings) and put $f\Lambda/\sqrt{2} = M_R$. Thus finally we obtain the following off-diagonal mass matrix for light neutrinos [14]

$$M_\nu = \frac{v^2}{2M_R}\begin{pmatrix} 0 & h_1^{(2)}h_2^{(3)} & h_1^{(2)}h_3^{(1)} \\ h_1^{(2)}h_2^{(3)} & 0 & h_2^{(3)}h_3^{(1)} \\ h_1^{(2)}h_3^{(1)} & h_2^{(3)}h_3^{(1)} & 0 \end{pmatrix} \tag{15}$$

The Yukawa couplings here are arbitrary and different choices for them provide different predictions. We shall consider two choices, called A and B, with different mass hierarchies. For the choice A we assume that the Yukawa couplings are proportional to the generation index of quarks

$$\begin{aligned} h_1^{(2)}\frac{v}{\sqrt{2}} &= \frac{1}{K}m_u \\ h_2^{(3)}\frac{v}{\sqrt{2}} &= \frac{1}{K}m_c \\ h_3^{(1)}\frac{v}{\sqrt{2}} &= \frac{1}{K}m_t \end{aligned} \tag{16}$$

where K is dimensioless parameter. Further we take [9] $m_u : m_c : m_t = \lambda^6 : \lambda^4 : 1$ as an order of magnitude relations. Then the mass matrix (15) can be written as

$$M_\nu = m_0\begin{pmatrix} 0 & \lambda^6 & \lambda^2 \\ \lambda^6 & 0 & 1 \\ \lambda^2 & 1 & 0 \end{pmatrix} \tag{17}$$

where

$$m_0 = \frac{\lambda^4 m_t^2}{K^2 M_R}. \qquad (18)$$

In the first approximation, it has eigenvalues $m_0\,(\pm 1, 0)\,[m_2 \simeq -m_3]$. The right-hand sides of Eqs. (6), (7) and (8) respectively become 1, λ^6 and λ^2. Eqs. (7) and (8) can then only be satisfied if both θ_2 and θ_3 are small so that $\sin\theta_{2,3} \simeq \theta_{2,3}$ while Eqs. (5) and (6) are also then satisfied for $\sin 2\theta_1 \simeq 1$, $\cos 2\theta_1 \simeq 0$, $\sin\theta_1 \simeq 1/\sqrt{2}$, $\cos\theta_1 \simeq 1/\sqrt{2}$. Writing Eqs. (6) and (7) in detail we have then

$$\begin{aligned} -\frac{\theta_2}{\sqrt{2}} + \frac{1}{2}\frac{1}{\sqrt{2}}\sin 2\theta_3 &= -\lambda^6 \\ -\frac{\theta_2}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}}\sin 2\theta_3 &= -\lambda^2 \end{aligned} \qquad (19)$$

implying

$$\sin 2\theta_3 \simeq \sqrt{2}\lambda^2\left(1-\lambda^4\right) \qquad (20)$$

$$\sin 2\theta_2 \simeq 2\theta_2 = \sqrt{2}\lambda^2\left(1+\lambda^4\right) \qquad (21)$$

Finally from Eq. (4), $m_2 \simeq -1$, $m_1 \simeq 0$ so that $m_3 \simeq 1$. In fact diagonalization of the matrix (17) give

$$\begin{aligned} m_3 &\approx m_0\left(\sqrt{1+\lambda^4}+\lambda^8\right) \\ m_2 &\approx m_0\left(-\sqrt{1+\lambda^4}+\lambda^8\right) \\ m_1 &\approx -2\lambda^8 m_0 \end{aligned} \qquad (22)$$

so that $m_3 \simeq |m_2| \gg |m_1|$ and

$$\Delta m_{12}^2 = \Delta m_{13}^2 = m_0^2, \quad \Delta m_{23}^2 \simeq 4 m_0^2 \lambda^8. \qquad (23)$$

Finally from Eqs. (2) and (3) to leading orders in s_2 and s_3

$$\begin{aligned} \nu_1 &\simeq \nu_e - s_3\left(c_1\nu_\mu - s_1\nu_\tau\right) - s_2\left(s_1\nu_\mu + c_1\nu_\tau\right) \\ \nu_2 &\simeq s_3\nu_e + c_3\left(c_1\nu_\mu - s_1\nu_\tau\right) \\ \nu_3 &\simeq s_2\nu_e + c_2\left(s_1\nu_\mu + c_1\nu_\tau\right) \end{aligned} \qquad (24)$$

showing that ν_1 is primarily ν_e while ν_2 and ν_3 are primarily $(c_1\nu_\mu - s_1\nu_\tau)$ and $(s_1\nu_\mu + c_1\nu_\tau)$ respectively.

We now consider the choice B, where we assume $h_1^{(2)} \gg h_1^{(3)} \simeq h_2^{(3)}$ and use the parametrization

$$\begin{aligned} h_2^{(3)} &= h\cos\theta, \quad h_1^{(3)} = h\sin\theta \\ \frac{h_1^{(3)}h_2^{(3)}}{h_1^{(2)}} &= h\sigma, \quad m_0 = \frac{h h_1^{(2)} v^2}{2M_R} \end{aligned} \qquad (25)$$

where $\sigma \ll 1$. Then

$$M_\nu = m_0\begin{pmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & 0 & \sigma \\ \sin\theta & \sigma & 0 \end{pmatrix} \qquad (26)$$

The diagonalization (neglecting σ^2) gives

$$\begin{aligned} m_{2,1} &= m_0\left[\pm 1 + \frac{1}{2}\sigma\sin 2\theta\right] \\ m_3 &= -\left(m_1+m_2\right) = -m_0\sigma\sin 2\theta \end{aligned} \qquad (27)$$

so that $|m_1| \simeq |m_2| \gg |m_3|$ and

$$\Delta m_{12}^2 = 2\sigma\sin 2\theta, \quad \Delta m_{31}^2 = \Delta m_{32}^2 = m_0^2 \qquad (28)$$

We will take $\theta_2 \simeq 0$ as before so that from Eq. (4), we must have $\theta_3 \simeq \frac{\pi}{4}$ in order to have $m_2 = -m_1$. Then from Eqs. (6), (7) and (8) in leading order, $\theta = -\theta_1$. In this case

$$\begin{aligned}
\nu_1 &\simeq \frac{1}{\sqrt{2}} \left[\nu_e - (\cos\theta_1 \nu_\mu - \sin\theta_1 \nu_\tau) \right] \\
\nu_2 &\simeq \frac{1}{\sqrt{2}} \left[\nu_e + (\cos\theta_1 \nu_\mu - \sin\theta_1 \nu_\tau) \right] \\
\nu_3 &\simeq (\sin\theta_1 \nu_\mu + \cos\theta_1 \nu_\tau) \qquad (29)
\end{aligned}$$

III. TRANSITION PROBABILITIES AND CONCLUSIONS

For our choice A, λ is expected to be of order $0.22 \simeq \sin\theta_c$ (θ_c being the Cabibbo angle) so that from Eqs. (20), (21) and (23)

$$\sin^2 2\theta_3 \simeq \sin^2 2\theta_2 \simeq 2\lambda^4 \simeq 4.5 \times 10^{-3} \qquad (30)$$

$$\Delta m_{23}^2 = 2 \times 10^{-5} m_0^2 \qquad (31)$$

Thus with

$$\Delta m_{12}^2 \simeq \Delta m_{31}^2 \gg \Delta m_{23}^2 \qquad (32)$$

and neglecting terms of order $s_3^4, s_2^2 s_3^2, \cos 2\theta_1 s_2 s_3$, we have from Eqs. (10)–(12)

$$\begin{aligned}
P(\nu_\mu - \nu_\tau)|_{\text{atm}} &= \sin^2 2\theta_1 \cos^2\theta_2 \cos^2\theta_3 \sin^2 \frac{\Delta m_{23}^2 R_a}{4E} \\
&\simeq \sin^2 2\theta_1 \sin^2 \frac{\Delta m_{23}^2 R_a}{4E} \qquad (33)
\end{aligned}$$

$$P(\nu_e - \nu_\mu)|_{\text{LSND}} = \left[\sin^2 2\theta_3 \cos^2\theta_1 + \sin 2\theta_1 \sin\theta_2 \sin 2\theta_3 + \sin 2\theta_2 \sin 2\theta_3 \sin\theta_1 \cos\theta_1 + \sin^2 2\theta_2 \sin^2\theta_1\right] \sin^2 \frac{\Delta m_{12}^2 R_{\text{LSND}}}{4E} \qquad (34)$$

and

$$\begin{aligned}
P(\nu_e - \nu_\tau) &= \left[\sin^2\theta_1 \sin^2 2\theta_3 - \sin 2\theta_1 \sin\theta_2 \sin 2\theta_3 - \sin\theta_1 \cos\theta_1 \sin 2\theta_2 \sin 2\theta_3 + \sin^2 2\theta_2 \cos^2\theta_1\right] \sin^2 \frac{\Delta m_{12}^2 R_s}{4E} \\
&+ \left[\sin\theta_1 \cos\theta_1 \sin 2\theta_2 \sin 2\theta_3\right] \sin^2 \frac{\Delta m_{23}^2 R_s}{4E} \qquad (35)
\end{aligned}$$

Now with $\theta_1 \simeq \frac{\pi}{4}$ and using Eq. (31) [note that the coefficient of $\sin^2\left(\Delta m_{12}^2 R_s/4E\right)$ in Eq. (35) vanishes], we have from Eqs. (34) and (35)

$$P(\nu_e \to \nu_\mu)|_{\text{LSND}} \simeq \sin^2 2\theta_{\text{eff}} \sin^2 \frac{\Delta m_{12}^2 R_{\text{LSND}}}{4E} \qquad (36)$$

$$P(\nu_e \to \nu_\tau)|_{\text{solar}} \simeq \sin^2 2\tilde{\theta}_{\text{eff}} \sin^2 \frac{\Delta m_{23}^2 R_s}{4E} \qquad (37)$$

where

$$\sin^2 2\theta_{\text{eff}} \simeq 2\,(4.5) \times 10^{-3} \simeq 10^{-2} \qquad (38)$$

$$\sin^2 2\tilde{\theta}_{\text{eff}} \simeq \frac{1}{2}\,(4.5) \times 10^{-3} = 2.25 \times 10^{-3} \qquad (39)$$

Thus with $m_0^2 \simeq 0.3$ eV^2, $\Delta m_{23} \simeq 4\lambda^8 m_0^2 \simeq 2 \times 10^{-5} m_0^2 \simeq 6 \times 10^{-6}$ eV^2 the LSND data and solar neutrino oscillations are explained, the latter with SMA MSW solution. Finally with $m_0 \simeq \sqrt{0.3}$ eV, $\lambda^4 = 4.5 \times 10^{-3}$ and $m_t \simeq 175$ GeV, we obtain from Eq. (18) $K^2 M_R \simeq 10^{11}$ GeV, giving the mass scale at which $e - \mu - \tau$ universality is broken and the

scale associated with superheavy Majorana neutrinos. In the version of the model we have considered $L_\tau - L_\mu - L_e$ number is , however, conserved while in the particular version of the Zee model mentioned earlier as well as in our model B, it is the $L_e - L_\mu - L_\tau$ number which is conserved.

We now consider the predictions of our version B for which

$$m_0^2 = \Delta m_{31}^2 = \Delta m_{32}^2 \gg \Delta m_{12}^2 \tag{40}$$

and $\theta_3 \simeq \frac{\pi}{4}$, $\theta_2 \simeq 0$ so that neglecting $\sin^2 2\theta_2$ and $\cos 2\theta_3$, we obtain in the leading order from Eqs. (10) and (13)

$$P\left(\nu_e \to \nu_\tau\right)\big|_{\rm atm} \simeq \sin^2 2\theta_1 \sin^2\left(\frac{\Delta m_{32}^2 R_a}{4E}\right) \tag{41}$$

$$P\left(\nu_e \to \nu_e\right)\big|_{\rm solar} \simeq 1 - \sin^2 2\theta_3 \sin^2\left(\frac{\Delta m_{12}^2 R_s}{4E}\right) \tag{42}$$

Thus atmospheric neutrino experimental data is explained with $\Delta m_{32}^2 \simeq m_0^2 \simeq 10^{-3}$ eV^2 and $\theta_1 \simeq \frac{\pi}{4}$ while the Eq. (42) is consistent with the large angle [$\sin^2 2\theta_3 \simeq 1$] vacuum or MSW solution in solar neutrino experiments. Here m_3, the mass of the lightest neutrino consisting mainly of ν_μ and ν_τ [cf. Eq. (29)] is given by

$$m_3 \simeq \sigma \sin 2\theta_1 m_0 = \sigma m_0 = \frac{\Delta m_{12}^2}{2m_0} = \frac{\Delta m_{\rm solar}^2}{2\sqrt{\Delta m_{\rm atm}^2}}.$$

To conclude by considering a simple extention of the standard model in which $(e - \mu - \tau)$ universality is not conserved, we have presented a scenario within the framework of see–saw mechanism in which the neutrino mass matrix is strictly off-diagonal in the flavor basis. Further we have shown that a version of this scenario can accomodate the atmospheric $\nu_\mu - \nu_\tau$ neutrino oscillations and large angle vacuum or MSW solution in solar neutrino experiments while another version is compatible with small angle MSW solution of solar neutrino oscillations and $\nu_\mu - \nu_e$ oscillations claimed by the LSND collaboration.

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FIG. 1. Majorana mass generation for light neutrinos